9.1 Introduction to Sequences

**Bellwork: 4/10/14**

Evaluate.

1. \((-1)^8\)  
2. \((11)^2\)  
3. \((-9)^3\)  
4. \((3)^2\)

Evaluate each expression for \(x = 4\).

5. \(2x + 1\)  
6. \(0.5x + 1.5\)

7. \(x^2 - 1\)  
8. \(2^x + 3\)

**Introduction to Sequences**

Find the \(n\)th term of a sequence.

Write rules for sequences.

Fibonacci considered the growth of an idealized (biologically unrealistic) rabbit population, assuming that:

- a newly born pair of rabbits, one male, one female, are put in a field;
- rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits;
- rabbits never die and a mating pair always produces one new pair once each month; and
- rabbits begin to produce offspring at the age of one month.

The puzzle that Fibonacci posed was: how many pairs will there be in any given month?

- At the end of the first month, there is only 1 pair.
- At the end of the second month, the female produces a new pair, so there are 2 pairs in the field.
- At the end of the third month, the original female produces another pair, making 3 pairs in all in the field.
- At the end of the fourth month, the original female produces her second pair, and the female born two months ago produces her first pair, making 5 pairs.
- At the end of the \(n\)th month, the number of pairs of rabbits is equal to the number of new pairs (which is the number of pairs in month \(n - 2\)) plus the number of pairs alive last month (month \(n - 1\)). This is the \(n\)th Fibonacci number.
Find the first 5 terms of the sequence with
\[ a_1 = -2 \text{ and } a_n = 3a_{n-1} + 2 \text{ for } n \geq 2. \]

\[ a_1 = 3(0) + 2 = 3 \cdot (-2) + 2 = -6 + 2 = -4, \]
\[ a_2 = 3(-4) + 2 = -12 + 2 = -10, \]
\[ a_3 = 3(-10) + 2 = -30 + 2 = -28, \]
\[ a_4 = 3(-28) + 2 = -84 + 2 = -82, \]
\[ a_5 = 3(-82) + 2 = -246 + 2 = -244. \]

Find the first 5 terms of the sequence.
\[ a_1 = -5, a_n = a_{n-1} - 8 \]
\[ a_2 = -5 - 8 = -13, \]
\[ a_3 = -13 - 8 = -21, \]
\[ a_4 = -21 - 8 = -29, \]
\[ a_5 = -29 - 8 = -37. \]

\[ a_n = 3n - 1. \]
\[ a_n = 3n - 5 \]

A ball is dropped and bounces to a height of 4 feet. The ball rebounds to 70% of its previous height after each bounce. Graph the sequence and describe its pattern.

How high does the ball bounce on its 10th bounce?

An ultra-low-flush toilet uses 1.6 gallons every time it is flushed.

Graph the sequence of total water used after \( n \) flushes, and describe its pattern. How many gallons have been used after 10 flushes?

A numerical sequence that shows the progression of notes (and rests).

Write a recursive formula and an explicit formula to generate this sequence.

In 4/4 time, a whole note represents 4 beats, a half note represents 2 beats, a quarter note represents 1 beat, and so on. Write a sequence for the number of beats that each note in the progression represents. Then write a recursive formula and an explicit formula to generate this sequence. How is this sequence related to the sequence in part a?)
Find the number of triangles in the 7th and 8th iterations of the Sierpinski triangle.